

Econ 204B: Section 9

Ryan Sherrard

University of California, Santa Barbara

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Endogenizing the Job Finding Rates

- ▶ We might want to be a little more formal with how we model the job finding rates
- ▶ Previously, we have assumed that agents get a job offer at some rate α
- ▶ Similarly, you may have noticed that we have essentially ignored the firm's side of the problem (workers draw from an exogenous wage dist.)
- ▶ Here we will think about both more seriously; we will put the “match” in “search and matching”

Matching Function

- ▶ Two things that are important when thinking about how easy it is to find a job are the number of unemployed persons (u) and the number of job vacancies (v)
- ▶ Diamond, Moretensen, and Pissarides: assume that the flow of contacts between firms and workers follows a matching technology $m = m(u, v)$
- ▶ Assuming that firms and workers are identical, the arrival rates of jobs are then given by

$$\underbrace{\alpha = \frac{m(u, v)}{v}}_{\text{for firms}} \quad \text{and} \quad \underbrace{\tilde{\alpha} = \frac{m(u, v)}{u}}_{\text{for workers}}$$

- ▶ $m(\cdot)$ is assumed to be continuous, nonnegative, increasing and concave in both arguments, with

$$m(u, 0) = m(0, v) = 0$$

- ▶ Another standard assumption is that $m(\cdot)$ is CRS: $m(cu, cv) = cm(u, v)$
 - ▶ increasing returns often yields multiplicity of equilibria
 - ▶ CRS is consistent with empirical results
- ▶ If, and when, we assume $m(\cdot)$ is CRS, the job finding probabilities only depend on $\theta \equiv v/u$, which is referred to as the *labor market tightness*

$$\alpha = \frac{m(u, v)}{v} = m\left(\frac{1}{\theta}, 1\right) \qquad \tilde{\alpha} = \frac{m(u, v)}{u} = m(1, \theta) = \theta\alpha$$

Firms

- ▶ A job is defined as a worker-firm pair
- ▶ A filled job is denoted by $J(\pi)$ where $\pi = y - w$ is profits (analogous to an employed worker)
- ▶ A vacant job is denoted by V (analogous to an unemployed worker)
- ▶ In order for a firm to post a vacancy, it must incur a flow cost $k > 0$ (a “recruitment cost”)
- ▶ Now let's consider a model with a separation rate of s and a job finding rate (from the firm's perspective) of α

Workers:

$$rU = b + \theta\alpha [V(w) - U]$$

$$rV(w) = w + s [U - V(w)]$$

Firms:

$$rV = -k + \alpha [J(\pi) - V]$$

$$rJ(\pi) = \pi + s [V - J(\pi)]$$

Number of Vacancies

- ▶ One loose-end that we have to hammer out regards how many firms post vacancies
- ▶ The standard way of doing so assumes a free-entry condition: firms enter until the value of a vacancy $V = 0$
- ▶ In other words, firms will enter, diluting the probability with which a firm may find a worker, until the value of posting that vacancy is 0

Wage Determination

- ▶ We still have yet to specify how wages are determined in this model; before we assumed an exogenous wage distribution $F(w)$
- ▶ One way of endogenizing the wage is to model w as the result of a (Nash) bargaining process
- ▶ The *generalized Nash bargaining* solution with *threat points* U (for workers) and V (for firms) is given by

$$w \in \operatorname{argmax} \left[\underbrace{V(w) - U}_{\text{worker surplus}} \right]^{\beta} \left[\underbrace{J(y - w) - V}_{\text{firm surplus}} \right]^{1-\beta}$$

- ▶ β is a parameter that captures the relative *bargaining power* of each party

The solution will satisfy the following.

$$\beta [J(y - w) - V] V'(w) = (1 - \beta) [V(w) - U] J'(y - w)$$

If we recall the expressions from earlier, we can very easily solve for the above derivatives.

$$V'(w) = J'(y - w) = \frac{1}{r + s}$$

Plugging this in ...

$$V(w) = U + \beta \left[\underbrace{J(y - w) - V + V(w) - U}_s \right].$$

The above states that the worker receives her threat point U and some share of the *surplus*:

$$S \equiv J(y - w) - V + V(w) - U.$$

We can use our earlier expressions for $J(\pi)$ and $V(w)$ to plug into the above:

$$S = \frac{y - rU - rV}{r + s}.$$

As we have done in the past, we can solve for the *reservation* strategies of workers and firms. Denote them w_R and π_R , respectively. We can easily determine

$$V(w) - U = \frac{w - w_R}{r + s} \quad \text{and} \quad J(\pi) - V = \frac{\pi - \pi_R}{r + s}.$$

The Nash bargaining problem can be simplified to

$$w \in \operatorname{argmax} [w - w_R]^\beta [y - w - \pi_R]^{1-\beta},$$

which can be solved ...

$$w = w_R + \beta(y - \pi_R - w_R).$$

- ▶ Importantly, notice that $w \geq w_R$ iff $y \geq y_R \equiv \pi_R + w_R$
- ▶ Similarly, $\pi = y - w \geq \pi_R$ iff $y \geq y_R$
- ▶ That is, workers and firms agree to form a relationship iff $y \geq y_R$
 - ▶ in other words, a relationship is formed if the match will produce enough for both the worker and firm to make a gain

Equilibrium

- ▶ An equilibrium here will be value functions ($J, V(w), U$), a wage w , and unemployment / vacancy rates (u, v)
- ▶ Let's look for a steady-state; from last class we know how to easily find the s.s. unemployment rate:

$$u = \frac{s}{\theta\alpha + s}$$

- ▶ Notice that there is no $[1 - F(w_R)]$ in the above statement
 - ▶ first, there is no exogenous wage distribution
 - ▶ second, with bargaining and assuming that matches are beneficial for both parties, whenever someone matches the wage will at least be as high as her reservation wage

- ▶ Rewrite the equation giving the surplus as follows

$$(r + s)S = y - rU$$

- ▶ Further, we can rewrite the flow value of unemployment utilizing the result from the bargaining process (that the worker will be paid her threat point plus a fraction of the surplus)

$$rU = b + \theta\alpha\beta S$$

- ▶ Plug this in to the first equation above

$$(r + s + \theta\alpha\beta)S = y - b \quad (\star)$$

- ▶ For the firm side of things, bargaining implies

$$J(\pi) = (1 - \beta)S$$

- ▶ Recalling that with free entry we have $V = 0$, we can rewrite the value of a job and plug in the above result

$$\theta\alpha J(\pi) = k \quad \implies \quad \alpha(1 - \beta)S = k \quad (\star)$$

- ▶ Both starred equations completely characterize the equilibrium; indeed, they may be combined

$$\frac{r + s + \theta\alpha\beta}{(1 - \beta)\alpha} = \frac{y - b}{k} \quad (\star\star)$$

- ▶ Since we know what u is, $(\star\star)$ may be solved for v (recall that v is embedded in θ and α):

$$\theta = \frac{v}{u} \quad \alpha = m\left(\frac{1}{\theta}, 1\right)$$

- ▶ Similarly, we'll be able to solve for a wage:

$$w = y - (r + s)(1 - \beta)S$$