Econ 204A: Section 1

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Preliminaries

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  Wednesday 2:00-3:00

Problem Sets:

- They will be due in class (not section) on Thursdays
- Grades will be a very simple "check" or "check minus"
- Problem sets should be TeX'd up. Learning LaTeX is an important skill, and will be useful come prelim time. (template on website)

Text Books / References (not required):

- Advanced Macroeconomics, 4th Ed. (Romer)
- Recursive Methods in Economic Dynamics (Stokey, Lucas, & Prescott)
- Recursive Macroeconomic Theory, 3rd Ed. (Ljungqvist & Sargent)

Overview of Econ 204A

- There are many things macroeconomists study: **growth**, inequality, business cycles, market frictions etc.
- Indeed, there is a lot of overlap in a lot of these broad topics
  - unifying theme: we think about the above in the context of aggregate variables / economies
- Important for many macro models is the savings decision of individuals / households (i.e. how much should I consume now vs. in the future)
- This class looks at a few models that describe this savings decision
  - in particular, we will look at the interplay between savings and growth

Some Terminology

- Representative Household: sometimes we are not focused on distributions of allocations in our models and so we can simplify our analysis to focus on one household that represents the economy as a whole
- Social Planner: this is a thought exercise wherein all choices are dictated (i.e. no market mechanism) by an all powerful agent who is trying to maximize the benefit in the economy
- Pareto Optimality: a notion of optimality in economics whereby no agent can be made better off without making another worse (by construction the social planner's solution is Pareto Optimal)
- Euler Equation: broadly, these describe the evolution of economic variables along an optimal path (these usually represent intertemporal / dynamic relationships)

The Models

1. The Solow Growth Model (exogenous growth)
2. The Ramsey Model (optimal growth / control)
3. The Diamond Model (overlapping generations)

- Within these models we will see a few applications and extensions of the basic theory
- Before we get to the meat of the course, though, let's review a simple intertemporal choice problem

A Two-Period Model

Consider a two-period model where agents maximize the utility function

$$U(c, c') = u(c) + \beta u(c'),$$

where $\beta < 1$ and $du(c)/dc > 0$. Agents receive income y in the first period, and y' in the second period, which can be split between consumption and savings, s, which earns interest at an exogenous rate r. Last, the government levies lump sum taxes t in the first period and t' in the second period.

(a) Write down the period-by-period budget constraints and derive the intertemporal budget constraint (IBC).

(b) Set up a Lagrangian and write down the first order conditions (FOCs).

(c) Assume agents discount at the rate of interest. Combine the FOCs and the IBC to find the optimal level of consumption in each period.

(d) Suppose $r = 0$. What does your answer from **(c)** suggest about consumption in each period?

(a) Write down the period-by-period budget constraints and derive the intertemporal budget constraint (IBC).

1st period: $c + s = y - t$

2nd period: $c' = y' - t' + (1+r)s$

$$
\begin{aligned}
\text{IBC:} \quad & s = y - t - c && \text{(from 1st period)} \\
\longrightarrow \quad & c' = y' - t' + (1+r)[y - t - c] && \text{(plug into 2nd period)} \\
\longrightarrow \quad & c + \frac{c'}{1+r} = y - t + \frac{y' - t'}{1+r} && \text{(reorganizing)}
\end{aligned}
$$

(b) Set up a Lagrangian and write down the first order conditions (FOCs).

$$\mathcal{L} = u(c) + \beta u(c') + \lambda \left[y - t + \frac{y' - t'}{1 + r} - c - \frac{c'}{1 + r} \right]$$

$$\frac{\partial \mathcal{L}}{\partial c} = u_c(c) - \lambda = 0$$
$$\frac{\partial \mathcal{L}}{\partial c'} = \beta u_c(c') - \frac{\lambda}{1 + r} = 0$$

Note: λ, the Lagrange multiplier, is often referred to as a shadow price. It is the instantaneous change in the value of the objective function that comes from a relaxation of the budget constraint ("by one unit").

(c) Assume agents discount at the rate of interest. Combine the FOCs and the IBC to find the optimal level of consumption in each period.

Agents discount at the rate of interest means that $\beta = \frac{1}{1+r}$. Then we can plug in β, solve for λ in one of the FOCs, and then plug that into the other FOC.

$$\lambda = u_c(c) \quad \longrightarrow \quad \frac{u_c(c)}{1+r} = \frac{u_c(c')}{1+r} \quad \Longleftrightarrow \quad \underbrace{u_c(c) = u_c(c')}_{\text{Euler Eqn.}}$$

Because $u(\cdot)$ is increasing everywhere, we must have that $c = c'$. Knowing this, we can plug in this optimality condition into the budget constraint and solve for c (or c').

$$c = c' = \frac{1+r}{2+r}\left[y - t + \frac{y' - t'}{1+r}\right]$$

(d) Suppose $r = 0$. What does your answer from **(c)** suggest about consumption in each period?

If $r = 0$, we can see that

$$c = c' = \frac{1}{2}\left[y + y' - t - t'\right],$$

meaning that agents will consume half of their lifetime income in each period.

Basics of The Solow Growth Model

- This is a very basic and mechanical model of growth (1950s)
- Not micro-founded: preferences are entirely captured by an exogenous savings rate
- This savings rate feeds directly into the dynamics of capital (which will determine the dynamics of other variables)
- The equilibrium concept we employ is the steady state, a notion of equilibrium wherein certain variables are constant over time
- Procedurally, to solve the model we will . . .
  1. transform variables into effective units
  2. derive the dynamics of capital
  3. solve for the steady state
  4. do some analysis

1. transform variables into effective units

- If X is an aggregate variable, define effective units as

$$x = \frac{X}{AL},$$

  where we normalize the aggregate variable per efficiency unit of labor. We do this in order to transform our variables into something stationary

- The model assumes a CRS production function, F, which allows us to do the following

$$Y = F(K, AL) \quad \implies \quad F\left(\frac{K}{AL}, \frac{AL}{AL}\right)$$

$$\implies \quad y = F(k, 1)$$

$$\implies \quad y = f(k)$$

2. derive the dynamics of capital

- Using the definition of k, take logs of both sides

$$k = \frac{K}{AL} \qquad \Longrightarrow \qquad ln(k) = ln(K) - ln(A) - ln(L)$$

- Though we typically suppress the notation, all of these variables are functions of time: $x(t)$. Using the above expression, take the derivative w.r.t. t

$$\frac{\dot{k}}{k} = \frac{\dot{K}}{K} - \frac{\dot{A}}{A} - \frac{\dot{L}}{L}$$

- From here we can build the rest of the model. We know that productivity and the labor force grow at rates g and n, respectively. Further, agents save a constant fraction s of output, and capital depreciates at rate δ. It follows that

$$\dot{K} = sY - \delta K$$

- Plug in those last results into the expression after we took the time derivative

$$\frac{\dot{k}}{k} = \frac{sY - \delta K}{K} - g - n$$

$$\frac{\dot{k}}{k} = \frac{sy}{k} - \delta - g - n$$

$$\dot{k} = \underbrace{sf(k)}_{\text{savings}} - \underbrace{(\delta + n + g)k}_{\text{break-even investment}}$$

3. solve for the steady state

- The functional form we usually assume for F is Cobb-Douglas, with exponents that sum to 1

$$F(K, AL) = K^{\alpha}(AL)^{1-\alpha} \qquad \Longrightarrow \qquad f(k) = k^{\alpha}$$

- The steady state is a equilibrium concept wherein variables of interest aren't moving over time. For us this is when $\dot{k} = 0$

$$0 = sk^{*\alpha} - (\delta + n + g)k^{*}$$

$$sk^{*\alpha} = (\delta + n + g)k^{*}$$

$$k^{*1-\alpha} = \frac{s}{\delta + n + g}$$

$$k^{*} = \left(\frac{s}{\delta + n + g}\right)^{\frac{1}{1-\alpha}}$$

- We can then solve for the rest of the steady state objects

Solow Analysis: Comparative Statics

- The first thing we might concern ourselves with is how our equilibrium changes when model parameters change
- We should (almost) always start with the equilibrium condition and take a total derivative to isolate the effect the change has on k^* (of which all other quantities are integrally related)

$$sf(k^*) = (\delta + n + g)k^*$$

$$f(k^*)ds + sf'(k^*)dk^* = (\delta + n + g)dk^* + (d\delta + dn + dg)k^*$$

- From here we can pick-and-choose which statics to look at (and then try to "sign" them)
- Let's walk through an example where we determine how k^*, y^*, and c^* are affected by a change in s

- The first thing we'll need to do is isolate the changes we want using the total derivative.

$$f(k^*)ds + sf'(k^*)dk^* = (\delta + n + g)dk^* + (0 + 0 + 0)k^*$$

$$\implies \qquad \frac{dk^*}{ds} = \frac{f(k^*)}{(\delta + n + g) - sf'(k^*)} > 0$$

- Now, we can "build up" to determine the effects on y^* and c^*.

$$y^* = f(k^*) \qquad \implies \qquad \frac{dy^*}{ds} = f'(k^*)\frac{dk^*}{ds} > 0$$

$$c^* = (1 - s)y^* \qquad \implies \qquad \frac{dc^*}{ds} = (1 - s)\frac{dy^*}{ds} - y^* \gtreqless 0$$

The Golden Rule and Dynamic Efficiency

- The closest notion of welfare here in the Solow model is consumption
  - we don't really care about output or capital insofar as they enable us to consume
- If we have the iconic Solow diagram in mind, we can see that if the s were really low/high then the intersection of $sf(k)$ and $(\delta + n + g)k$ will be very close to $f(k)$ (i.e. consumption is low)
- Because of the shape of our curves, there is some middle ground where consumption is maximized
- That level is called the **Golden Rule** level of k
- It's the level you would have wanted people in the past to have chosen

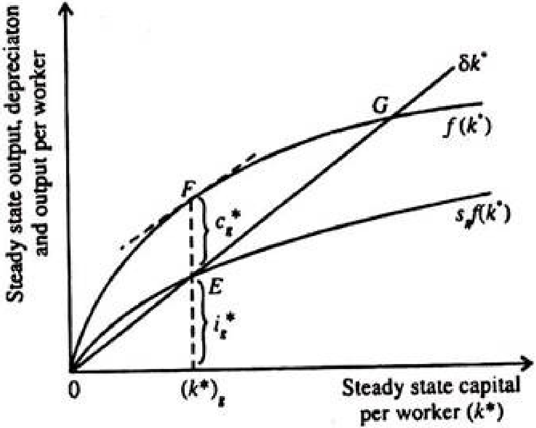

Fig. 4.8 The Saving Rate and the Golden Rule

- How will we determine k^{gr}? Well, we know that it will be chosen such that $dc/dk = 0$

$$c = f(k) - (\delta + n + g)k \qquad \Longrightarrow \qquad \frac{dc}{dk} = f'(k) - (\delta + n + g) = 0$$

$$f'(k^{gr}) = (\delta + n + g)$$

- I.e. it is the level of k such that the output curve is tangent to the break-even line

- Under the usual Cobb-Douglas specification with capital's share $= \alpha$, we'll have

$$k^{gr} = \left(\frac{\alpha}{\delta + n + g}\right)^{\frac{1}{1-\alpha}}$$

- Compare the Golden rule level of k with the general steady state value of k we found before:

$$k^{gr} = \left(\frac{\alpha}{\delta + n + g}\right)^{\frac{1}{1-\alpha}} \qquad \text{vs.} \qquad k^* = \left(\frac{s}{\delta + n + g}\right)^{\frac{1}{1-\alpha}}$$

- Immediately we can see that $k^* = k^{gr}$ if $s = \alpha$. That is, we are saving in the same rate as capital's return per unit of output.

- If the economy is in a position where $s > \alpha$ (i.e. we are saving more than what we are getting back in capital returns), then the economy is said to be **dynamically inefficient**

- In such a scenario, it would be a Pareto improvement to immediately lower s (allowing greater consumption right away and into the future)

- Is this true when $s < \alpha$? No. An increase in s would be required which has negative effects on consumption today (though c will be higher in the LR)

General Tips for this Quarter / Year

- Hit the ground running; feeling behind is normal (even ubiquitous)
- Get on a schedule; make some time for fun
- Work together; those who are solo acts tend to do worse (or are overworked)
- Don't think about prelims yet; focus on this quarter
- At this point you guys know what works for you better than others; be cautious of snake oil salesmen!

More Specific Tips for Macro

- Stay organized and really think about the big picture
- Understand what each step in a solution does, and how to combine these steps to get to a solution
- You will frequently get a lot of answers that look like "alphabet soup"
- The complexity that arises from simple models just goes to show how complex economics is
- More so, those expressions are important (not life altering, but still)
  - we will be able to answer more difficult questions
  - you can only get so far with "supply goes up, demand goes down"